Fermi Questions

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When Jack Cairns originally asked me to provide a write-up describing the event, **Fermi Questions**, I was confronted with the same sort of dilemma that was expressed to me by a coach, "How do I teach for the event?". I jumped at the opportunity since I believe that **Fermi Questions** represent some of the most favorable features of the Science Olympiad. After considerable internal debate (i.e., talking to myself), I've come to the conclusion that this event is a 'culmination event', that is, it uses the knowledge, logic, and critical thinking concepts, developed during a person's lifetime, to attack and solve problems. **Fermi Questions** are essentially problems requiring the correct order of magnitude as the solution which otherwise is either too difficult to measure or whose answer is imprecise.

I have been supervising **Fermi Questions** as a state event in Delaware for the past twenty years or so. It has been especially rewarding to me to watch how well the students (generally, teams of two) collaborate to solve the problems. Knowing how much effort I expend in making the exam, I am quite gratified to watch these budding scientists expend their mental energies in kind. For that reason, I try to make the questions fun, a learning experience, and relevant to their quest for knowledge. Each year, I provide copies of the **Fermi Questions** event to a half dozen university professors who then administer it to their students (both graduate and undergraduate) as a learning/teaching exercise.

In the course of making up the event, I consider several kinds of questions:

* math (straight) – where the answer can be calculated using a calculator or computer but, since such aids are not allowed in the competition, it forces the student to consider other routes to provide a reasonable answer

* how answers from one problem relate to other problems – as with many facets of life, an answer to one problem leads to many other choices and problems.

* having solutions to problems relate to 'real life', for example, a problem might ask for an estimate of the amount of gasoline used by passenger cars in the U.S., how an increase in gas mileage would relate to a decrease in green-house gas production, and how the amount of water produced by same relates to other items such as rainfall or filling of swimming pools.

In short, if something has numbers associated with it, that subject is fair game for a **Fermi Question**.

Underlying Considerations

* Behind each problem set that I create is the tacit assumption that the contestants have a reasonable knowledge of mathematics, specifically, the use and operations of exponential notation. The lack of math skills is not too apparent when the answers to the problems are in the range 0.001 to 1000 (**Fermi Question** notation –3 to +3). But when I ask the students to calculate the number of iron atoms on the head of a pin, the inability to handle exponents readily shows (there are approximately $3*10^{13}$ iron atoms – **Fermi Question** answer +13; see Example xiv. below). I can't count the number of times that I've seen students cover the scrap paper (that I distribute for them to use in their deliberations) with zeroes. For that reason, it is imperative to stress the use of exponential notation (which also serves as the

basis for the metric system). Not only does the use of exponential notation permit calculations to be done faster, it also helps avoid problems with writing, transcribing, and counting the correct number of zeroes. So much so, that in some branches of science there are specially named units that have very large (or very small) numbers associated with them, such as, one Angstrom = 10^{-8} cm, one Light Year = $5.9*10^{12}$ miles, Avogadro's Number = $6.023*10^{23}$.

* As I noted previously, an important component of the event is logical, critical thinking. Reading and understanding the problem is one important component; the other important component is to develop a plan to provide the answer in the requested units.

* And finally, time is a critical parameter. The ability to think and calculate rapidly can be learned – the keywords are, in the immortal words of a Hall-of-Fame football coach, "practice, practice, and more practice". I have watched students (when I was a coach) significantly lower the times required to solve these problems. In fact, some of them have returned from college and told me that the same skills (required to solve **Fermi Questions**) permitted them to handle tests and problem sets much faster than their contemporaries.

Typically, the first time that a team tries to solve a problem, they try to be too exact. For example, if the **Fermi Question** is "how many toothpicks are equivalent to the perimeter of Colorado?", they discuss the length of a toothpick ("is it 2.0, 2.25, 2.45 inches?); then they try to estimate the perimeter of the state; and finally, they calculate a value. Any time there is a discussion, time is lost. Since the answer to any question is the correct order of magnitude, an error of a factor of two or three will still probably yield the correct exponent (the **Fermi Question** answer). Hence, they should pick a value and work up their answer. The time that they save will be needed to solve other problems.

Below is the presentation that I delivered at the October, 2003 Coaches Clinic in Hammond, Indiana. Almost all of the attendees were very positive about the event as I presented it; several State Directors, in attendance, told me that they were definitely going to have **Fermi Questions** as a State Event. I sent them the questions (with all work shown and answers) that I used in the 2004 Delaware competition. If your State would like me to assist with preparing the **Fermi Quest**ion event, please e-mail me at Lloyd.Abrams@USA.DuPont.com. As I noted previously in this article, I have supervised the **Fermi Event** in Delaware for over 20 years. I've kept most of the problem sets and I will provide them to you as a tutorial for the kinds of questions that might be asked (contact me via e-mail).

Got a **Fermi Question**? If you, one of your event supervisors, coaches, or students, have what they think will be a good **Fermi Question**, by all means please forward it to me.

Presentation at the Science Olympiad Coaches Clinic, Hammond, Indiana, Oct, 2003.

Fermi Questions is named after Enrico Fermi, a Nobel Laureate in Physics, who was famed for being able to do order-of-magnitude calculations in his head. For example, after watching the first atomic bomb explosion, he immediately calculated that the strength of the explosion was equivalent to the explosion of 20 kilotons of TNT. It took another three weeks for a panel of the Manhattan Project's best scientific brains to do an 'exact' calculation; the answer that they came up with, yes, you guessed it - 20 kilotons. Such calculations are sometimes called 'ballpark estimates' or 'back-of-the-envelope' calculations. While these calculations were important, years ago, because one had to keep track of decimal places when using a slide rule, these calculations are still very important because an approximate answer will often dictate the amount of resources required to attack a problem. For example, when you ask a wedding consultant to plan the affair, they often ask the question, "How many people will attend the dinner?". Your approximate answer will allow them to estimate the amount of food required, the number of tables and their layout, the size of the hall to be rented, etc., etc. Fundamental to the solution of these problems is a skill called Critical Thinking - essentially a method of attacking such problems in an orderly, logical way. This skill can be learned and it is the underlying basis for the event.

Why this event? Numbers (when you think about it) are a measure of our surroundings and life.

Examples

- how many air molecules are in this room (where I was presenting this lecture)?
- how many pounds of CO_2 and H_2O does the U.S. population expel in a year?
- how many tons of food are consumed in Chicago during the course of a day?
- how many people are involve in delivering and preparing that food?
- how many gallons of paint do you need to paint the walls of your school?
- how many baseballs are used during the course of a Major League season?
- how many pizzas were eaten last year in the U.S.?

The scoring for the event is like horseshoes:
5 points for the correct exponent
3 points for the correct exponent ± 1
1 point for the correct exponent ± 2

The answer to a **Fermi Question** is the correct exponent of 10 (if an answer is $5*10^n$, round the answer up to the next power of 10; I try to manage the problems so that answers are not $5*10^n$). Generally, if a team averages 3 points per problem and there are 30 problems, the 90 points that they will have achieved will garner them a medal. Calculators, computers, or any other device, including crib sheets, lists of constants, formulae, etc., are not permitted. All the contestants need are pencils (with erasers) and a good night's sleep - I supply scratch paper (to simulate the 'back-of-envelopes'). Positive exponential values are the default; negative exponents MUST have the - (minus) sign as part of the answer.

Some considerations involved when learning to solve these problems:

1. **Exponents are short-hand notation** (knowledge of which makes it easier and faster to solve the problems). The notation used below is: Ex. = example; Ans. = Answer; FA = Fermi Answer.

Ex. What is the population of New York City? Ans. 7,000,000 = $7*10^6 \sim 10^7$ FA **7**
Ex. What is the distance, in miles, from the Earth to the Sun? Ans. 100,000,000 = 10^8 FA **8**

2. **Properties of exponents**.

$500 = 5*10^2$; 5 is the coefficient, 10 is the base, 2 is the exponent
when multiplying, add exponents of the same base
Ex. $200*4000 = 2*10^2 * 2^2*10^3 = 2^3*10^5 = 8*10^5 \sim 10^6$ FA **6**
when dividing, subtract exponents of the same base
Ex. $200 \div 800 = 2*10^2 \div 2^3*10^2 = 2^{-1}*10^0 = 2^{-1}*1 \sim 10^{-1}$ FA **-1**
note that $10^0 = 1$
$10^{20} = (10^4)^5 = (10^2)^{10}$; $2^{10} \sim 10^3$

3. **Round off values BEFORE doing a calculation**. This makes it much easier and faster to do the problems. Why? because the FA is the correct order of magnitude which means that there is a large range that yields the correct answer. For example, the FA for the distance, in miles, from the Earth to the

Sun is 8 (shown previously in 1.) but the range of values giving the same answer is $5*10^7$ to $4.99*10^8$!!
In this context, I suggest using the values below which are somewhat different from the exact values:

Item	Exact Value	Fermi Value (for ease of calculation)
1 day	24 hours	25 hours
1 mile	5280 feet	5000 feet
1 yard	0.9144 meter	1 meter
1 foot	30.48 cm	30 cm
1 pound	453.6 g	500 g
1 hour	3600 seconds	4000 seconds

4. **Always keep the units as part of working a problem**. In some instances, keeping track of the units will lead to the correct answer. I am particularly sensitive to the use of units since I have degrees in both engineering (British units are used, pounds, feet, BTU, etc.) and chemistry (metric units, grams, meters, calories, etc.) AND the U.S. uses both of these systems. Sometimes the units get left off solutions to real problems with tragic, unforeseen results. As an example, most US cooks know what a $^1/_4$ pound of butter looks like - it is a stick about 1 inch x 1 inch x 5 inches. But ask them what 100 g of butter looks like and they may throw up their hands in defeat. The answer is that the stick is almost the same size since 100 g is close to $^1/_4$ pound.

5. **What subject matter is covered**? Everything is fair game! If the item in question has numbers associated with it, I might use it. In the past, I have given questions on math, chemistry, physics, biology, geology, geography, economics, swimming, basketball, running, census, food, waste generation, ...

Examples. (abbreviation: F?s = Fermi Question solution) These can be done by the students as practice; have them show all work and what assumptions they made in solving the problems.

i. How many seconds are there in a year?
Exact solution: 60 sec/min * 60 min/hr * 24 hr/day * 365 day/year = $3.15*10^7$ s/y FA 7
F?s: 4000 s/h * 25 h/d * 400 d/y = $4*10^3 * 2.5*10^1 * 4*10^2 = 4*2.5*4*10^6 = 4*10^7$ FA 7

Note that both answers are the same. Budding Fermi Question experts should remember that there are $3*10^7$ seconds in a year - this will probably save them time in solving another F?.

ii. How many miles are there in a light-year?
Exact solution: 186,000 m/s * $3.15*10^7$ s/y = $1.86*3.15*10^{12} = 5.9*10^{12}$ m/y FA 13
F?s: $2*10^5$ m/s * $3*10^7$ s/y = $6*10^{12}$ m/y

This quantity is a basic unit used in astronomy. As noted in problem i., knowing that there are $3*10^7$ seconds in a year has shortened the work considerably.

iii. How many kilometers are there in a light-year?
F?s: $6*10^{12}$ mi/y * 1.6 km/mi = $10*10^{12}$ km/y = 10^{13} FA 13

iv. For the average American woman, how many times will her heart beat during her lifetime?
Assumptions: 1 heartbeat per second, lifetime of 80 years
F?s: $3*10^7$ s/y * 1 hb/s * 80 y/lifetime = $2.4*10^9$ FA 9

v. How many heartbeats are there in a year for the entire world's population?
Assumptions: 1 heartbeat per second, $6*10^9$ people
F?s = $3*10^7$ s/y * 1hb/s * $6*10^9$ = $18*10^{16} = 1.8*10^{17}$ FA 17

vi. How many pounds of rice were consumed in the U.S. in the year 2001?

Assumptions: 20 pounds of rice eaten per year by a person, $3*10^8$ people in the U.S.

F?s: 20 #/p * $3*10^8$ p = $6*10^9$ # = 10^{10} FA 10

This answer was checked using data from the U.S. Dept. of Agriculture; $5.2*10^9$ lbs. If the students assume 2-10 or 200-1000 #/p, they would still get 3 points.

vii. What is the density of butter in g/cc?

Assumptions: 1 pound of butter is a package 2 inch x 2 inch x 5 inches.

F?s: V = 2 in * 2.5 cm/in * 2 in * 2.5 cm/in * 5 in * 2.5 cm/in = 5 * 5 * 12 = 300 cm^3

Density = M/V = 500 g / 300 cm^3 = 1.5 g/cm^3 ~ 1 = 10^0 FA 0

viii. What fraction of a mile is a cm?

F?s: 1 cm / (5000 f/mi * 30 cm/f) = 1 / $3*5*10^4$ = 1 / $1.5*10^5$ = 10^{-5} FA -5

ix. What is the area, in sq. miles, of the original 48 states of the U.S.?

Assumption: the U.S. is shaped like a rectangle; 3000 miles wide x 1000 miles

F?s: $3*10^3$ * $1*10^3$ = $3*10^6$ FA 6

Note: when areas are requested, it is much easier to use a rectangle.

x. What is the area of the U.S. (prob. ix.) in cm^2?

F?s: $3*10^6$ mi^2 * 1.6^2 km^2/mi^2 * $(10^3)^2$ m^2/km^2 * $(10^2)^2$ cm^2/m^2

= 3*2.5 * $10^{(6+6+4)}$ = 7.5 * 10^{16} ~ 10^{17} FA 17

xi. What is the area of Lake Superior in sq. miles?

Assumption: the lake is shaped like a rectangle; 300 miles wide x 100 miles

F?s: $3*10^2$ mi * $1*10^2$ mi = $3*10^4$ FA 4

xii. Estimate the volume of Lake Superior in cubic kilometers.

V = Area * Depth; Assumption: Average depth is 200 m

F?s: $3*10^4$ mi^2 * 1.6^2 km^2/mi^2 * 200 m * 1 km/1000 m

= 3 * 2.5 * 2 * $10^{(4+2-3)}$ = 15 * 10^3 = $1.5*10^4$ FA 4

xiii. How many cubic kilometers of rain fall of the U.S.(48 states) in one year? Assume an average rainfall of 10 inches.

F?s: $3*10^6$ mi^2 * 1.6^2 km^2/mi^2 * 10 in * 2.5 cm/in * 1 m/10^2 cm * 1 km/10^3 m

= 3*2.5*2.5 * $10^{(6+1-2-3)}$ = 20 * 10^2 = 2 * 10^3 FA 3

Note that Lake Superior has 7 times the total volume of rainfall: the Great Lakes have about half of the Earth's fresh water.

xiv. How many iron atoms are on the head of a pin?

Assumption: the head is 1 mm in diameter, diameter of an iron atom is 2.5 Angstroms

Area of the head of a pin: $^1/_2$*pi*D^2 = 1.5 * $(1 \text{ mm} * 1 \text{ cm}/10 \text{ mm})^2$ = 1.5 * 10^{-2} cm^2

Assume that the head of a pin is half a sphere

Area covered by an iron atom: $^1/_4$*pi*D^2 = 0.75 * $(2.5*10^{-8})^2$ cm^2 = 5 * 10^{-16} cm^2

F?s: $1.5*10^{-2}$ cm^2 / $5*10^{-16}$ cm^2 = $0.3*10^{14}$ = 3 * 10^{13} FA 13

Note: this problem can also be solved using the rectangle approach.

Assumption: the head is 1 mm on a side, an iron atom is 2.5 Angstroms on a side

Area of the head of a pin: $^1/_2$*6*S^2 = 3 * $(1 \text{ mm} * 1 \text{ cm}/10 \text{ mm})^2$ = 3 * 10^{-2} cm^2

Area covered by an iron atom: S^2 = $(2.5*10^{-8})^2$ cm^2 = 6.25 * 10^{-16} cm^2

F?s: $3*10^{-2}$ cm^2 / $6.25*10^{-16}$ cm^2 = $0.4*10^{14}$ = 4 * 10^{13} FA 13